جلسه سوم حل تمرین درس ترمودینامیک

منبع : کتاب ون وایلن ، امتحانات دانشگاه واترلو

12.72

A tank has two sides initially separated by a diaphragm. Side A contains 1 kg of water and side B contains 1.2 kg of air, both at 20°C, 100 kPa. The diaphragm is now broken and the whole tank is heated to 600°C by a 700°C reservoir. Find the final total pressure, heat transfer and total entropy generation.

C.V. Total tank out to reservoir.

Energy Eq.5.11: $U_2 - U_1 = m_a(u_2 - u_1)_a + m_v(u_2 - u_1)_v = {}_1Q_2$

Entropy Eq.8.14 and 8.18:

$$S_2 - S_1 = m_a(s_2 - s_1)_a + m_v(s_2 - s_1)_v = {}_1Q_2/T_{res} + S_{gen}$$

Volume: $V_2 = V_A + V_B = m_v v_{v1} + m_a v_{a1} = 0.001 + 1.009 = 1.01\ m^3$

$v_{v2} = V_2/m_v = 1.01,\ T_2 \Rightarrow P_{2v} = 400$ kPa

$v_{a2} = V_2/m_a = 0.8417,\ T_2 \Rightarrow P_{2a} = mRT_2/V_2 = 297.7$ kPa

$P_{2tot} = P_{2v} + P_{2a} =$ **697.7 kPa**

Water table B.1: $u_1 = 83.95$ kJ/kg, $u_2 = 3300$ kJ/kg,

$s_1 = 0.2966$ kJ/kg K, $s_2 = 8.4558$ kJ/kg K

Air table A.7: $u_1 = 293$ kJ/kg, $u_2 = 652.3$ kJ/kg,

$s_{T1} = 2.492$ kJ/kg K, $s_{T2} = 3.628$ kJ/kg K

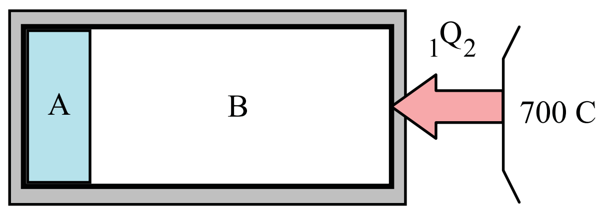
From energy equation we have

${}_1Q_2 = 1(3300 - 83.95) + 1.2(652.3 - 293) =$ **3647.2 kJ**

From the entropy equation we have

$S_{gen} = 1(8.4558 - 0.2966) + 1.2[3.628 - 2.492 - 0.287\times\ln(301.6/100)]$

$- 3647.2 / 973.2 =$ **5.4 kJ/K**



$$c_i = \frac{m_i}{m_{tot}} = \frac{n_i M_i}{\sum n_j M_j} = \frac{n_i M_i / n_{tot}}{\sum n_j M_j / n_{tot}} = \frac{y_i M_i}{\sum y_j M_j} \quad (12.3)$$

and from a mass basis to a mole basis as

$$y_i = \frac{n_i}{n_{tot}} = \frac{m_i / M_i}{\sum m_j / M_j} = \frac{m_i / (M_i m_{tot})}{\sum m_j / (M_j m_{tot})} = \frac{c_i / M_i}{\sum c_j / M_j} \quad (12.4)$$

For the Dalton model of gas mixtures, the properties of each component of the mixture are considered as though each component exists separately and independently at the temperature and volume of the mixture, as shown in Fig. 12.2. We further assume that both

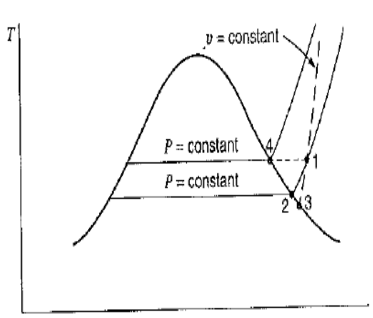
$$R_{mix} = \frac{1}{m}\left(\frac{PV}{T}\right) = \frac{1}{m}(n\overline{R}) = \overline{R}/M_{mix}$$

$$S = ms = m_A s_A + m_B s_B$$
$$= m(c_A s_A + c_B s_B)$$

$$R_{mix} = \frac{1}{m}(n_A\overline{R} + n_B\overline{R})$$
$$= \frac{1}{m}(m_A R_A + m_B R_B)$$
$$= c_A R_A + c_B R_B$$

$$(s_2 - s_1)_i = s_{0i} - s_{0i} + C_{p0i}\left[\ln\frac{T_2}{T_0} - \ln\frac{T_1}{T_0}\right] - R_i\left[\ln\frac{y_i P_2}{P_0} - \ln\frac{y_i P_1}{P_0}\right]$$
$$= 0 + C_{p0i}\ln\left[\frac{T_2}{T_0} \times \frac{T_0}{T_1}\right] - R_i \ln\left[\frac{y_i P_2}{P_0} \times \frac{P_0}{y_i P_1}\right]$$
$$= C_{p0i}\ln\frac{T_2}{T_1} - R_i \ln\frac{P_2}{P_1}$$

The relative humidity ϕ is defined as the ratio of the mole fraction of the vapor in the mixture to the mole fraction of vapor in a saturated mixture at the same temperature and total pressure. Since the vapor is considered an ideal gas, the definition reduces to the ratio of the partial pressure of the vapor as it exists in the mixture, P_v, to the saturation pressure of the vapor at the same temperature, P_g:



$$\phi = \frac{P_v}{P_g}$$

$$\phi = \frac{P_1}{P_4}$$

$$\omega = \frac{m_v}{m_a}$$

$$\omega = \frac{P_v V/R_v T}{P_a V/R_a T} = \frac{R_a P_v}{R_v P_a} = \frac{M_v P_v}{M_a P_a}$$

:ure, this reduces to

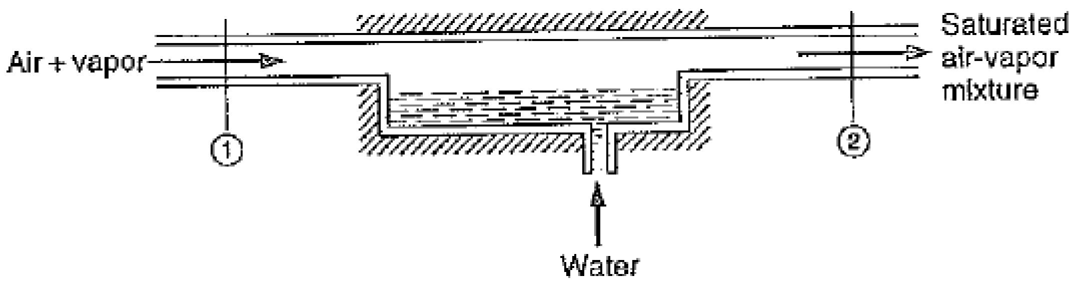
$$\phi = \frac{\omega P_a}{0.622\, P_g}$$

$$\omega = 0.622 \frac{P_v}{P_a}$$

$$h_{a1} + \omega_1 h_{v1} + (\omega_2 - \omega_1) h_{l2} = h_{a2} + \omega_2 h_{v2}$$

$$\omega_1 (h_{v1} - h_{l2}) = C_{pa}(T_2 - T_1) + \omega_2 (h_{v2} - h_{l2})$$

$$\omega_1 (h_{v1} - h_{l2}) = C_{pa}(T_2 - T_1) + \omega_2 h_{fg2}$$



The processes that take place at the wet-bulb thermometer are somewhat complicated. First, if the air–water vapor mixture is not saturated, some of the water in the wick evaporates and diffuses into the surrounding air, which cools the water in the wick. As soon as the temperature of the water drops, however, heat is transferred to the water from both the air and the thermometer, with corresponding cooling. A steady state, determined by heat and mass transfer rates, will be reached, in which the wet-bulb thermometer temperature is lower than the dry-bulb temperature.

12.66

A mixture of 60% helium and 40% nitrogen by mass enters a turbine at 1 MPa, 800 K at a rate of 2 kg/s. The adiabatic turbine has an exit pressure of 100 kPa and an isentropic efficiency of 85%. Find the turbine work.

Solution:

Assume ideal gas mixture and take CV as turbine.

Energy Eq.6.13: $w_{T\,s} = h_i - h_{es}$,

Entropy Eq.9.8: $s_{es} = s_i$, adiabatic and reversible

Process Eq.8.32: $T_{es} = T_i(P_e/P_i)^{(k-1)/k}$

Properties from Eq.12.23, 12.15 and 8.30

$C_{P\,mix} = 0.6\times 5.193 + 0.4\times 1.042 = 3.5326$ kJ/kg K

$R_{mix} = 0.6\times 2.0771 + 0.4\times 0.2968 = 1.365$ kJ/kg K

$(k-1)/k = R/C_{P\,mix} = 1.365/3.5326 = 0.3864$

$T_{es} = 800(100/1000)^{0.3864} = 328.6$ K

$w_{Ts} = C_P(T_i - T_{es}) = 3.5326(800 - 328.6) = 1665$ kJ/kg

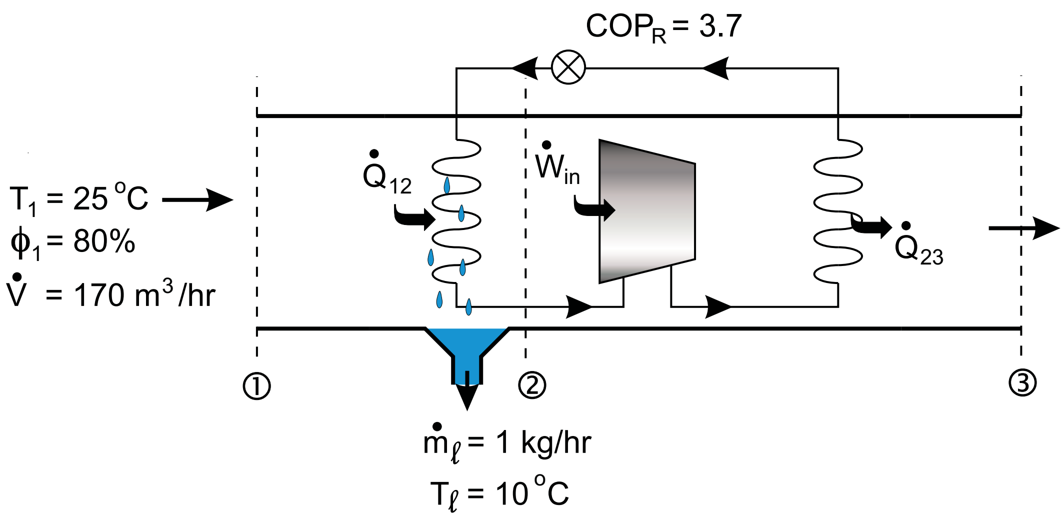
$w_{T\,ac} = \eta w_{Ts} = 1415.5$ kJ/kg

$\dot{W}_{T\,ac} = \dot{m} w_{T\,ac} =$ **2831 kW**

Question 4 (20 marks)

A common household dehumidifier operates using a vapour-compression refrigeration process. In the first section, moist air is cooled and dehumidified by extracting heat through the refrigeration system's evaporator coil. The condensed water is removed at $\mathbf{10\ °C}$. In the second section, the moist air is reheated by adding heat through the refrigeration system's condenser coil. The unit removes water at a rate of $\mathbf{1\ kg/hr}$ when the inlet conditions are $\boldsymbol{T_1 = 25\ °C}$, $\boldsymbol{\phi_1 = 80\%}$ and $\boldsymbol{\dot{V}_1 = 170\ m^3/hr}$. The COP of the refrigeration system is $\boldsymbol{COP_R = 3.7}$. If the moist air is at a constant pressure of $\mathbf{1\ atm}$ throughout the system, determine:

a) the temperature and specific humidity at state 2
b) the work input [$\boldsymbol{kW}$] to the compressor
c) the temperature and relative humidity at state 3



Part a)

We can obtain the specific volume of the air at the inlet conditions from the psychrometric chart

$$\nu_1 = 0.866\ m^3/kg_a$$

The mass flow rate of the air is given as

$$\dot{m}_a = \frac{\dot{V}_1}{\nu_1} = \frac{170\ m^3/hr}{0.866\ m^3/kg_a} = 196.3\ kg_a/hr$$

Performing a mass balance over the section **1 − 2** gives

$$\dot{m}_{w1} = \dot{m}_{w2} + \dot{m}_\ell$$

$$\dot{m}_{w2} = \dot{m}_{w1} - \dot{m}_\ell$$

Dividing through by the mass of air gives

$$\begin{aligned} \omega_2 &= \omega_1 - \frac{\dot{m}_\ell}{\dot{m}_a} \\ &= 0.016\ \frac{kg_w}{kg_a} - \frac{1\ kg_w/hr}{196.3\ kg_a/hr} \\ &= 0.0109\ \frac{kg_w}{kg_a} \Leftarrow \end{aligned}$$

Knowing that state point 2 is a saturated liquid, we can get the temperature from the psychrometric chart

$$T_2 = 15.5\ ^\circ C \Leftarrow$$

Part b)

Determine the amount of heat removed from the coil between state **1 − 2**.

We can state that

$$\dot{m}_a = \dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3}$$

Performing an energy balance over this section gives

$$\dot{m}_a h_1^* = \dot{m}_a h_2^* + \dot{Q}_{12} + \dot{m}_\ell h_\ell$$

From the psychrometric chart

$$\begin{aligned} h_1^* &= 66\ kJ/kg_a \\ h_2^* &= 43.5\ kJ/kg_a \end{aligned}$$

From the saturation tables

$$h_\ell = 42.01\ kJ/kg_w$$

Therefore

$$\begin{aligned}\dot{Q}_{12} &= \dot{m}_a(h_1^* - h_2^*) - \dot{m}_\ell h_\ell \\ &= (196.3\ kg_a/hr)(66 - 43.5\ kJ/kg_a) - (1\ kg_w/hr)(42.01\ kJ/kg_w) \\ &= 4374.74\ kJ/hr = 1.21\ kW\end{aligned}$$

The work into the compressor can be determined as

$$\dot{W}_{in} = \frac{\dot{Q}_{12}}{COP_R} = \frac{4374.74\ kJ/hr}{3.7} = 1182.36\ kJ/hr = 0.328\ kW \Leftarrow$$

Part c)

To determine the outlet conditions we need to find the heat flow rate into the reheat section. Performing an energy balance over the refrigeration loop

$$\dot{W}_{in} + \dot{Q}_{12} = \dot{Q}_{23}$$

Therefore

$$\dot{Q}_{23} = 1182.36\ kJ/hr + 4374.74\ kJ/hr = 5557.1\ kJ/hr$$

Performing an energy balance between $2-3$ gives

$$\dot{m}_a h_2^* + \dot{Q}_{23} = \dot{m}_a h_3^*$$

$$\begin{aligned}h_3^* &= h_2^* + \frac{\dot{Q}_{23}}{\dot{m}_a} \\ &= 43.5\ kJ/kg_a + \frac{5557.1\ kJ/hr}{196.3\ kg_a/hr} \\ &= 71.8\ kJ/kg_a\end{aligned}$$

Knowing that for a heating process, $\omega_3 = \omega_2 = 0.0109\ kg_w/kg_a$, we can get from the psychrometric chart

$$\begin{aligned}T_3 &= 43.2\ ^\circ C \Leftarrow \\ \phi_3 &= 20\% \Leftarrow\end{aligned}$$

12.7

A rigid container has 1 kg argon at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any external heat transfer. What is final T, P? Is any s generated?

Energy Eq.: $U_2 - U_1 = 0 = 2mu_2 - mu_{1a} - mu_{1b} = mC_v(2T_2 - T_{1a} - T_{1b})$

$$T_2 = (T_{1a} + T_{1b})/2 = \mathbf{350\ K},$$

Process Eq.: V = constant =>

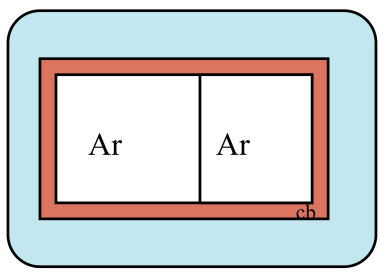
$$P_2V = 2mRT_2 = mR(T_{1a} + T_{1b}) = P_1V_{1a} + P_1V_{1b} = P_1V$$

$$P_2 = P_1 = \mathbf{150\ kPa},$$

ΔS due to temperature changes only, not P, internally we have a Q over a ΔT

$$\Delta S = m\,(s_2 - s_{1a}) + m\,(s_2 - s_{1b}) = mC_p\,[\,\ln (T_2/T_{1a}) + \ln (T_2/T_{1b})\,]$$

$$= 1 \times 0.520\left[\ln\frac{350}{300} + \ln\frac{350}{400}\right] = \mathbf{0.0107\ kJ/K}$$



12.8

A rigid container has 1 kg CO_2 at 300 K and 1 kg argon at 400 K both at 150 kPa. Now they are allowed to mix without any heat transfer. What is final T, P?

No Q, No W so the energy equation gives constant U

Energy Eq.: $U_2 - U_1 = 0 = m_{CO2}(u_2 - u_1)_{CO2} + m_{Ar}(u_2 - u_1)_{Ar}$

$$= m_{CO2}C_{v\ CO2}(T_2 - T_1)_{CO2} + m_{Ar}C_{v\ Ar}(T_2 - T_1)_{Ar}$$

$$= (1\times0.653 + 1\times0.312) \times T_2 - 1\times0.653\times300 - 1\times0.312\times400$$

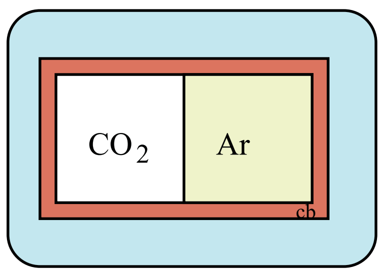
$T_2 =$ **332.3 K**,

$$V = V_1 = V_{CO2} + V_{Ar} = m_{CO2}R_{CO2}T_{CO2}/P + m_{Ar}R_{Ar}T_{Ar}/P$$

$$= 1\times0.1889\times300/150 + 1\times0.2081\times400/150 = 0.932\ 73\ m^3$$

Pressure from ideal gas law and Eq.12.15 for R

$P_2 = (1\times0.1889 + 1\times0.2081) \times332.3/0.932\ 73 =$ **141.4 kPa**



12.53

Two insulated tanks A and B are connected by a valve. Tank A has a volume of 1 m^3 and initially contains argon at 300 kPa, 10°C. Tank B has a volume of 2 m^3 and initially contains ethane at 200 kPa, 50°C. The valve is opened and remains open until the resulting gas mixture comes to a uniform state. Determine the final pressure and temperature.

Solution:

C.V. Tanks A + B. Control mass no W, no Q.

Energy Eq.5.11: $U_2 - U_1 = 0 = m_{Ar}C_{V0}(T_2 - T_{A1}) + m_{C_2H_6}C_{VO}(T_2 - T_{B1})$

$m_{Ar} = P_{A1}V_A/RT_{A1} = (300 \times 1) / (0.2081 \times 283.15) = 5.0913$ kg

$m_{C_2H_6} = P_{B1}V_B/RT_{B1} = (200 \times 2) / (0.2765 \times 323.15) = 4.4767$ kg

Continuity Eq.: $m_2 = m_{Ar} + m_{C_2H_6} = 9.568$ kg

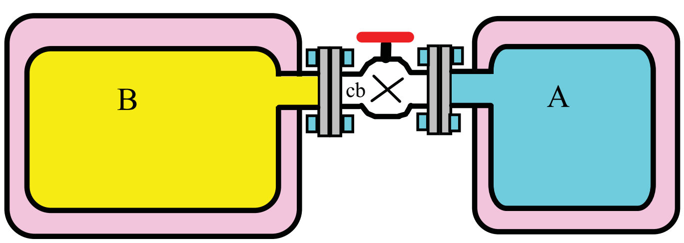
Energy Eq.: $5.0913 \times 0.312\ (T_2 - 283.2)$

$+ 4.4767 \times 1.490\ (T_2 - 323.2) = 0$

Solving, $T_2 =$ **315.5 K**

$$R_{mix} = \Sigma\, c_i R_i = \frac{5.0913}{9.568} \times 0.2081 + \frac{4.4767}{9.568} \times 0.2765 = 0.2401 \text{ kJ/kg K}$$

$P_2 = m_2RT_2/(V_A+V_B) = 9.568 \times 0.2401 \times 315.5/3 =$ **242 kPa**



12.57

A 50/50 (by mass) gas mixture of methane CH_4 and ethylene C_2H_4 is contained in a cylinder/piston at the initial state 480 kPa, 330 K, 1.05 m^3. The piston is now moved, compressing the mixture in a reversible, polytropic process to the final state 260 K, 0.03 m^3. Calculate the final pressure, the polytropic exponent, the work and heat transfer and entropy change for the mixture.

Solution:

Ideal gas mixture: CH_4, C_2H_4, 50% each by mass => $c_{CH_4} = c_{C_2H_4} = 0.5$

$$R_{mix} = \Sigma\, c_i R_i = 0.5 \times 0.5183 + 0.5 \times 0.2964 = 0.40735 \text{ kJ/kg K}$$

$$C_{v\ mix} = \sum c_i\, C_{vi} = 0.5 \times 1.736 + 0.5 \times 1.252 = 1.494 \text{ kJ/kg K}$$

State 1: $m = P_1V_1/R_{mix}T_1 = 480 \times 1.05 / (0.40735 \times 330) = 3.7493$ kg

State 2: $T_2 = 260$ K, $V_2 = 0.03$ m^3, Ideal gas PV = mRT so take ratio

$$=> \quad P_2 = P_1 \frac{V_1}{V_2}\frac{T_2}{T_1} = \mathbf{13\ 236\ kPa}$$

Process: PV^n = constant and PV = mRT gives $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$,

$$\ln\frac{T_2}{T_1} = (n-1)\ln\frac{V_1}{V_2} \quad => \quad \mathbf{n = 0.933}$$

also for this process we get Eq.8.38 or Eq.4.4

$$=> \qquad {}_1W_2 = \int P\, dV = \frac{1}{1-n}(P_2V_2 - P_1V_1) = \mathbf{-1595.7\ kJ}$$

Energy Eq.: $\quad {}_1Q_2 = U_2 - U_1 + {}_1W_2 = m\, C_{v\ mix}\,(T_2 - T_1) + {}_1W_2$

$$= 3.7493 \times 1.494(260 - 330) - 1595.7 = \mathbf{-1988\ kJ}$$

Change of entropy from Eq.8.26

$$s_2 - s_1 = C_{v\ mix} \ln(T_2 / T_1) + R_{mix} \ln(V_2 / V_1)$$

$$= 1.494 \ln(260 / 330) + 0.40735 \ln(0.03 / 1.05)$$

$$= \mathbf{-1.8045\ kJ/kg\ K}$$

and

$$S_2 - S_1 = m(s_2 - s_1) = 3.7493\,(-1.8045) = \mathbf{-\ 6.7656\ kJ/K}$$

12.43

A piston/cylinder device contains 0.1 kg of a mixture of 40 % methane and 60 % propane gases by mass at 300 K and 100 kPa. The gas is now slowly compressed in an isothermal (T = constant) process to a final pressure of 250 kPa. Show the process in a P-V diagram and find both the work and heat transfer in the process.

Solution:

C.V. Mixture of methane and propane, this is a control mass.

Assume methane & propane are ideal gases at these conditions.

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Property from Eq.12.15

$$R_{mix} = 0.4\ R_{CH4} + 0.6\ R_{C3H8}$$
$$= 0.4 \times 0.5183 + 0.6 \times 0.1886 = 0.3205 \text{ kJ/kg K}$$

Process: T = constant & ideal gas =>

$${}_1W_2 = \int P\,dV = mR_{mix}T \int (1/V)dV = mR_{mix}T \ln (V_2/V_1)$$
$$= mR_{mix}T \ln (P_1/P_2)$$
$$= 0.1 \times 0.3205 \times 300 \ln (100/250) = \mathbf{-8.81\ kJ}$$

Now heat transfer from the energy equation where we notice that u is a constant (ideal gas and constant T) so

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = {}_1W_2 = \mathbf{-8.81\ kJ}$$